# ON THE CHORD COMMON TO A PARABOLA AND THE CIRCLE OF CURVATURE AT ANY POINT.

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It is known that if a circle meet a parabola in four points, the sum of the distances of the points on one side of the axis from it is equal to the sum of the distances of the points on the other side from it. If three of the points are coincident, the circle becomes the circle of curvature, and the distance of the three coincident points ($P$) from the axis is one-third of that of the fourth point from the axis.

Hence the common chord of the circle and parabola is divided by the axis in the ratio 1 : 3. But the shorter segment of the chord is equal to the tangent at $P$, since they are equally inclined to the axis. Therefore the chord is equal to four times the tangent. Let $y^2 = 4ax$ be the equation to the parabola, and $(x', y')$ the co-ordinates of $P$. Then

$$y - y' = -\frac{2a}{y'}(x - x'), \quad \text{or} \quad yy' + 2ax - \tfrac{3}{2}y'^2 = 0,$$

is the equation to the chord.

Differentiating with respect to $y'$, $y = 3y'$; hence $y^2 = -12ax$ is the envelope of the chord. Also, from relation $y = 3y'$, it follows that the longer segment of the chord is equal to the corresponding tangent of the parabola, $y^2 = -12ax$.

The point $P$, and the point where the chord prolonged touches $y^2 = -12ax$, are harmonic conjugates with respect to the points where it meets the axis and the tangent at the common vertex of the parabolas.

The tangent at the end of the *latus rectum* of $y^2 = -12ax$ is normal to $y^2 = 4ax$ at the end of its *latus rectum*, and therefore touches its evolute. The chord is then a diameter of the circle of curvature, and is bisected by its point of contact with the evolute.

Hence the radius of curvature = twice the normal = $4a\sqrt{2}$, which agrees with a known result.